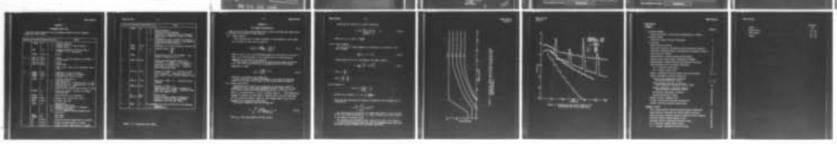
AD-A068 375 WEAPONS SYSTEMS RESEARCH LAB ADELAIDE (AUSTRALIA) F/G 20/1
A RAY TRACING MODEL FOR PROPAGATION LOSS CALCULATIONS. (U)
AUG 78 D V WYLLIE
UNCLASSIFIED WSRL-0035-TM NL

| OF |
AD
A0 375



END
DATE
FILMED
6 - 79
DDC

LEVEL II

WSRL-0035-TM

AR-001-382

ADA068375

DDC FILE COPY

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

WEAPONS SYSTEMS RESEARCH LABORATORY

DEFENCE RESEARCH CENTRE SALISBURY
SOUTH AUSTRALIA

TECHNICAL MEMORANDUM

WSRL-0035-TM

A RAY TRACING MODEL FOR PROPAGATION LOSS CALCULATIONS

D.V. WYLLIE

Technical Memoranda are of a tentative nature, representing the views of the author(s), and do not necessarily carry the authority of this Laboratory.

Approved for Public Release

COPY No. 15

UNCLASSIFIED

AR-001-382

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

WEAPONS SYSTEMS RESEARCH LABORATORY

LEVEL II

9 TECHNICAL MEMORANDUM

14 WSRL-0035-TM

6

A RAY TRACING MODEL FOR PROPAGATION LOSS CALCULATIONS.

10 D.V. Wyllie

11 Aug 78

12 19p.

SUMMARY

A brief description is given of a ray tracing model developed for making propagation loss predictions for use in underwater acoustic research. A range independent sound speed profile and ocean depth are assumed. The various options available to the user are discussed together with some results, which highlight the importance of having available good bottom loss versus grazing angle data. A FORTRAN computer programme is currently available for use on the IBM 370/168 at Defence Research Centre Salisbury.

Approved for public release

Technical Memoranda are of a tentative nature, representing the views of the author(s), and do not necessarily carry the authority of this Laboratory.

POSTAL ADDRESS: Chief Superintendent, Weapons Systems Research Laboratory
Box 2151, G.P.O., Adelaide, South Australia, 5001

410 919

UNCLASSIFIED

A

JB

DOCUMENT CONTROL DATA SHEET

Security classification of this page: UNCLASSIFIED

1 DOCUMENT NUMBERS	
AR Number:	AR-001-382
Report Number:	WSRL-0035-TM
Other Numbers:	

2 SECURITY CLASSIFICATION	
a. Complete Document:	UNCLASSIFIED
b. Title in Isolation:	UNCLASSIFIED
c. Summary in Isolation:	UNCLASSIFIED

3 TITLE

A RAY TRACING MODEL FOR PROPAGATION LOSS CALCULATIONS

4 PERSONAL AUTHOR(S):

D.V. Wyllie

5 DOCUMENT DATE:

August 1978

6 6.1 TOTAL NUMBER OF PAGES 18

6.2 NUMBER OF REFERENCES: 13

7 7.1 CORPORATE AUTHOR(S):

Weapons Systems Research Laboratory

7.2 DOCUMENT SERIES AND NUMBER

Weapons Systems Research Laboratory
0035-TM

8 REFERENCE NUMBERS

a. Task: DST 76/125

b. Sponsoring Agency: DST

9 COST CODE:

10 IMPRINT (Publishing organisation)

Defence Research Centre Salisbury

11 COMPUTER PROGRAM(S) (Title(s) and language(s))

12 RELEASE LIMITATIONS (of the document):

Approved for public release

12.0	OVERSEAS	NO		P.R.	1	A		B		C		D		E	

Security classification of this page: UNCLASSIFIED

Security classification of this page: UNCLASSIFIED

13 ANNOUNCEMENT LIMITATIONS (of the information on these pages):

No limitation

14 DESCRIPTORS:

a. EJC Thesaurus Terms

Mathematical models
Ray tracing
Beams (Radiation)
Propagation
Wave Propagation
Underwater acoustic
Acoustic scattering
Underwater sound transmission

b. Non-Thesaurus Terms

15 COSATI CODES:

2006
2001

16 LIBRARY LOCATION CODES (for libraries listed in the distribution):

SW SR SD AACA

17 SUMMARY OR ABSTRACT:
(if this is security classified, the announcement of this report will be similarly classified)

A brief description is given of a ray tracing model devloped for making propagation loss predictions for use in underwater acoustic research. A range independent sound speed profile and ocean depth are assumed. The various options available to the user are discussed together with some results, which highlight the importance of having available good bottom loss versus grazing angle data. A FORTRAN computer programme is currently available for use on the IBM 370/168 at Defence Research Centre Salisbury.

Security classification of this page: UNCLASSIFIED

TABLE OF CONTENTS

LIST OF APPENDICES

LIST OF FIGURES

1. INTRODUCTION

This paper describes the operation of, and some results from a computer programme used in propagation loss calculations for underwater acoustic research. Ray theoretical expressions are employed in the calculations and an ocean with a flat bottom and a range independent sound speed profile are assumed. The main part of the programme was written by Dr. H. Bucker of the Naval Ocean Systems Center (formerly the Naval Undersea Center), San Diego. The programme was adapted for use on the IBM 7090 and later on the IBM 370/168 at WSRL and has been in use for several years. During this time a number of modifications and additions have been made, which are described in this paper.

The programme was written to provide fast and accurate propogation loss estimates from a source to a number or receivers. It employs a ray sweep out method, which is described in Reference 1. This method identifies all significant ray paths from a source to a receiver during a single sweep out of the source beam patterns and at the same time calculates the total ray intensity at the receiver.

No attempt is made to describe the details of the programming techniques used. However, the input data required to use the program on the DRC IBM 370/168 are included in Appendix I. The names of the more important input variables are included as bracketed capitals, when the various options are being discussed in the text.

2. APPROXIMATING THE SOUND SPEED PROFILE

The sound speed profile used as input to this and many other propagation loss programmes usually consists of set of sound speed values at a number of specified depths. These values may have been read off a continuous trace of a sound speed profile or be the results of measurements at discrete depths (e.g., resulting from Nansen cast measurements of temperature and salinity). If the depth/sound speed values have been obtained from a continuous trace then the values are selected so that the important characteristics of the sound speed profile are retained (e.g. depth of mixed layer, depth of SOFAR axis etc.). It is important to keep the number of depth/sound speed values fairly small, since the computing time increases with the number of values employed; ten values are usually adequate unless the profile is unduly complicated.

Given a set of depth/sound speed values the programme offers three options for fitting a profile. A brief description will be given of these options.

2.1 Linear interpolation

The first is a linear interpolation (ITYPRO = 0) between the input depth/sound speed values. The profile is linear in the variable $1/C^2$, where C is the local sound speed and z is the depth;

$$1/C^2 = a + bz, \quad .... \qquad (1)$$

where a and b are constants. Use of this expression is common practice in acoustics, since it allows exact solutions to be found for the range travelled by a ray over a given depth interval (e.g. Appendix II and reference 2). Selection of this option results in a profile made up of segments satisfying equation (1) and passing through the given depth/sound speed values.

2.2 Natural spline

In order to eliminate spurious intensity values, which arise when the gradient of the sound speed profile is discontinuous(ref.3), a natural spline can be fitted through the sound speed/depth values using the second of the profile fitting options. The natural spline is a cubic polynomial;

$$1/C^2 = a + bz + cz^2 + dz^3, \ldots. \qquad (2)$$

which has continuous first and second derivatives, passes through the input data values and has zero valued second derivatives at the boundaries. The coefficients (a, b, c and d) are determined by an iterative numerical procedure for each segment of the profile. Care should always be exercised in the selection of the input data values so that the fitted curve is a reasonable replica of the original profile. This can be a problem where there are sudden changes in the sound speed/depth profile, e.g. at the bottom of the mixed layer or if step changes occur in the thermocline. The curve fitting procedure sometimes results in overshoot and an unacceptable profile. Use of alternative methods for calculating the spline (e.g. ref.4) may improve the fit; these have not been investigated.

2.3 Natural spline with least-squares fit

The third option available requires two sets of sound speed/depth data. A spline is fitted to the second set and the mean-square difference between values of $1/C^2$ determined from the spline and those from the first data set is computed. The $1/C^2$ values in the second set are then modified, a new spline fitted and the mean-square difference evaluated. This process continues until the mean-square difference is less than a specified value. This option has not been used in any of the studies using the programme, but may be useful if several sets of data are available over an area of interest and a best fit needs to be found.

3. SURFACE IMAGING EFFECTS

It is well known(ref.5) that when an acoustic source is near the surface (i.e. within several wavelengths) the propagation loss to a distant receiver is increased over that for a deeper source. This arises because of interference between the surface reflected and direct ray paths and is sometimes referred to as surface decoupling. The effect is of special interest in calculating the radiated noise from surface ships, since they radiate principally at low frequencies and have an acoustic centre with a depth of only a fraction of a wavelength at these frequencies.

The programme does not include the facility of coherently summing the contributions from different ray paths. The intensity at a receiver is computed from the spacing of adjacent rays, which bracket a receiver (Appendix II) and the total intensity found by summing the intensities due to all ray pairs. In order to include the near-surface effects a near-surface source is replaced by a single source at the surface with a dipole radiation pattern (INVBP = 3);

$$F(\theta) = 4 \sin(2\pi\, d/C.f \sin\theta\ ), \ldots.. \qquad (3)$$

where θ is the ray elevation angle measured from the horizontal, f is the frequency, d is the source depth and C is the sound speed at the surface.

This is only an approximation to the real situation but should be quite good

when the near-surface water has a constant sound speed at least to the source depth.

A near-surface receiver is treated in a slightly different fashion and this is discussed in the next section.

4. RAY ARRIVAL STRUCTURE AND VERTICAL BEAM PATTERNS

As well as providing the total intensity at a receiver, using the technique outlined in Appendix II, the program can provide the vertical arrival structure at a receiver (IXPR Negative). Up to 100 angle increments can be employed over the vertical angle range 0° to 180° and the total intensity of all rays arriving at the receiver in each increment (added incoherently) can be determined.

The above option is necessary when it is required to compute the response of an array with some directivity in the vertical plane (IXPR = -2). The vertical beam pattern (beam in dB units) is provided as input data. The number of values must be less than or equal to the number of angle increments above. If less, the last given value of the beam pattern is assumed to hold over the remainder of the vertical angle range. The vertical beam pattern is then convolved with the acoustic arrival pattern and the array output for all steered directions of the array determined.

The above procedure is only approximate, since it assumes that the sound field is uniform over the vertical extent of the array, that is each ray is associated with a plane wave incident on the vertical array and the total acoustic field is the sum of the energies of these acoustic plane waves. The approximation will be quite good when the array is short, but will be less accurate when the array extends over a depth range across which there is a significant change in the sound field. This later case has to be treated by a more exact method as is done for example in reference 6, where the sound field amplitude and phase at each receiver in the array were computed using a mode theory model.

It is possible to input a vertical beam pattern for the source in addition to the two options, omnidirectional (INVBP = 0) and dipole (INVBP = 3), previously discussed. Using another option (INVBP = 1) the vertical pattern is entered at 1° intervals for the vertical angle range 0 to 180° and is assumed to be symmetrical about 0°. The final option (INVBP = 2) permits two sets of vertical patterns to be entered covering the range 0 to 180° and 0 to -180° at 1° intervals, where the angles are measured from the horisontal. It is not necessary to enter all 360 values; entering a value equal to or less than zero terminates the input list and automatically sets all subsequent data points to the last value read.

Another input parameter (NTILTD) permits these vertical beams to be steered above and below the horizontal. A positive value steers the main beam in a direction above the horisontal.

5. BOTTOM LOSS

A number of bottom loss options (IABOT) is available within the program. A perfectly absorbing bottom (IABOT = 2) perfectly reflecting bottom (IABOT = 0) or a bottom loss table given at 1° intervals (IABOT = 1) can be specified in the input data. If the value of the input variable (IABOT) lies between 11 and 18 a set of bottom loss versus grazing angle values are selected from a table contained in the subroutine, BMLOS. The set selected depends on the frequency and bottom roughness (IR equal to IABOT -10) specified. This table was taken from the FACT(ref.7) ray tracing model.

For frequencies between 1 and 3.5 kHz the bottom losses are those given by the proposed US Navy interim standard bottom loss curves. Eight roughness (IR = 1 to 8) are available for this frequency range. Sets of bottom loss

values are also given for the frequency intervals 150 Hz and below, 150 Hz to 700 Hz, 700 Hz to 1 kHz, 3.5 kHz to 4.5 kHz and for frequencies greater than 4.5 kHz. Five roughness options are available within each frequency range (IR = 1 to 5). Figure 1 shows the bottom loss versus grazing angle curves for frequencies less than 150 Hz for each roughness value.

Propagation loss is very sensitive to bottom loss and the selection of bottom-loss values versus grazing angle is a particularly difficult problem if reliable PL preductions are required.

Propagation loss predictions (figure 2) have been made using the programme for a frequency of 100 Hz, for two water depths (2000 m and 5000 m) and for two bottom roughnesses (IR = 1 and 4). The most striking feature is the large differences between the curves, even at quite modest ranges (e.g. 22 dB at 100 km).

The results for the 5000 m water depth provide a good example of the sensitivity of the PL results to the bottom loss data used. The major contributions to the received energy at ranges exceeding about 25 km arises from rays having grazing angles at the bottom less than about 15 degrees. Over this angular range the BL values used in generating the PL curves differ by only 2 to 5 dB. In order to make a prediction of PL accurate to several dB at 150 km the BL data must be known to better than 1 dB. Data with this accuracy are rarely available in Australian waters, especially at the lower frequencies.

The convergence zone peaks in figure 2 are of course independent of the BL. No correction is made in the model for the intensity of sound in the vicinity of a ray theory caustic as is done, for example, in the FACT model. The method used in the current program (Appendix II), which involves spatial averaging, usually results in modest intensity values within a convergence zone, and excessively high intensity values (or low PL) generally don't occur. The variations in the heights of the peaks in figure 2 are due to the finite range interval between successive PL determinations.

Results for the shallower water depth of 2000 m, show a greater divergence of PL values for the different BL data, because of the greater number of bottom bounces required to reach a given range. At 150 km the PL results differ by 19 dB compared to 10 dB in the deeper water. Consequently the accuracy of the BL data must be increased to between one quarter and one half dB if the PL preductions are going to be accurate to several dB at this range. In still shallower water, the problem would be worse except that the ranges of interest begin to be reduced because of the overall increase in the PL.

In addition to these problems associated with BL data accuracy there is another question relating to the adequacy of the description of the BL in terms of simple Rayleigh plane wave reflection coefficients at low frequencies(ref.8). There is considerable penetration of sound into the bottom. Associated with this penetration are reflection from sub-bottom interfaces, refraction in the sub-bottom layers, absorption in the layers, generation of shear waves in the solid layers and generation of surface waves at the layer interfaces, to mention only some of the bottom effects. The present model is only capable of using simple reflection coefficients; this may not be adequate at low frequencies in some areas.

A common feature of low frequency BL data (using reflection coefficient interpretation at small grazing angles(ref.8) is a negative loss due to refraction and possibly reflections within the bottom layers. BL data of this type were obtained in the north-eastern Indian Ocean during a recent survey conducted by the U.S. Naval Oceanographic Office. The preliminary results are given in reference 9.

In conclusion, the uncertainties in the BL data make PL predictions difficult for a particular area and reliable BL data must be available for areas of interest. Often the differences between two models become unimportant because of the basic uncertainties in the input data required by both.

6. ADDITIONAL FUNCTIONS

There are several additonal programme functions which should be mentioned:

(1) A surface reflection loss (SLDB) can be included but is independent of surface grazing angle.

(2) Volume absorption(ref.10) is included if a frequency value is provided in the input data(FREQ).

(3) Line-printer plot of PL versus range (IPRL).

7. CONCLUDING REMARKS

The programme has been quite useful for making a variety of PL and ray arrival structure predictions over the past few years. It has of course the limitation of dealing only with a single sound speed profile in a flat bottom ocean, but should continue to be useful for a wide range of applications. Current model developments at DRCS (e.g. ref.11) are concentrating on models which can handle range dependent ocean depth and sound speed profiles. These models have immediate use in two areas of interest, the first over the continental slope and the second in the vicinity of oceanographic fronts and eddies. However, they are demanding of computing time and must be used with some care, so that the current ray model should still find many uses.

REFERENCES

No.	Author	Title
1	Bucker, H.P.	"The Ray Sweep-out Method". JASA, 52, 283-286, 1972.
2	Pedersen, M.A., Gordon, D.F.	"Normal Mode Theory Applied to Short-Range Propagation in an Underwater Acoustic Surface Duct". JASA, 37, 105-118, 1965.
3	Pedersen, M.A.	"Acoustic Anomalies Introduced by Constant Velocity Gradients". JASA, 33, 465-474, 1961.
4	Muler, C.B., Solomon, L.P.	"Use of Splines and Numerical Integration in Geometrical Acoustics". JASA, 48, 739-744, 1970.
5	Pedersen, M.A.	"Surface Decoupling". Proceedings of International Workshop on Low Frequency Propagation and Noise, Woods Hole. October 1974.
6	Wyllie, D.V., Gordon, D.F.	"Theoretical Performance Loss of Vertical Arrays Due to Signal Propagation Effects". JUA, 25, No. 1, 77, 1975.
7	Spofford, C.W.	"The FACT Model Maury Center Report 109". AESD, ONR, Virginia, November 1974.
8	Santaniello, S., Dinapoli, R.R., Dullea, R.K. and Herstein, P.D.	"A Synopsis of Studies on the Interaction of Low Frequency Acoustic Signals with the Ocean Bottom". NUSC Tech. Doc. 5337, June 1976.
9	Wyllie, D.V.	"Project BOOLEE - Sea Test Measurements and Preliminary Results - 1975/76". WSRL-0015-TR May 1978
10	Urick, R.J.	"Principles of Underwater Sound for Engineers". McGraw Hill, p92, 1967.
11	Gartrell, G.	"A Parabolic Equation Propagation Loss Model". WSRL-0034-TR, August 1978.
12	Tolstoy, I., Clay, C.S.	"Ocean Acoustics". McGraw Hill, 1966.
13	Abramowitz, M., Stegun, I.A. (Ed)	"Handbook of Mathematical Function". US National Bureau of Standards, 1970 (p887).

APPENDIX I

PROGRAMME INPUT DATA

The data cards required to run the program together with the required formats are given below.

Card No.	Variable	Format	Options	Notes
1	ITYPRO	I2	0	Linear segments (see section 2)
			1	Natural spline (see section 2)
			2	Fitted spline (see section 2)
	NGQ	I2	3	Gaussian Quadrature variable used if ITYPRO = 1 or 2 (Section 2)
	TITLE	10A4		Title
2,3 ...	ZML (I)	F8.1		Depth } Only required if ITYPRO = 2
	VML (I)	F8.3		Sound speed } Only required if ITYPRO = 2
*	ZILD (I)	F8.1		Depth
	VIL (I)	F8.3		Sound speed
				Zero value of VML or VIL terminates input list
*A	GAMUD	F3.1		Upgoing ray limit at source
	DGAMD	F3.1		Ray angle increment for IPLRAY = 1
	GAMDD	F3.1		Downgoing ray limit at source
	INVBP	I1		Vertical beam pattern
			0	Omnidirectional
			1	Read pattern at 1° intervals for 0 to 180°
			2	Read pattern at 1° intervals for 0 to 180° followed by 0 to -180°
			3	Dipole source pattern (see sections 3 and 4)
	NTILTD	I2		Beam steer angle: positive for upward pointing beam
	IPLRAY	I2		Test function - not required
	SORLEV	F5.2		Source level (dB re ref. level)
	SLDB	F4.2		Surface loss (dB)
	IABOT	I2		Bottom loss
			0	Perfectly reflecting
			1	Requires loss table at 1° intervals
			2	Perfectly absorbing
			1X	$1 \leq X \leq 8$ Define bottom roughness (see section 5)
	IATT	I2		Not used
	INDSVP	I2		Not used
	RF	F10.0		Initial receiver range (metres or yards)
	DR	F10.0		Range increment (metres or yards)
	RL	F10.0		Final receiver range (metres or yards)

Card No.	Variable	Format	Options	Notes
	IXPR	I2	0 1 2 -1 -2	 Print diagnostic Extensive print diagnostic Intensity vs angle printout at spacing of DELT (see Sub TRACE) Max number of range pts is 40 Intensity vs angle followed by convolution with receiver beam pattern. Max number of range pts 40 (see section 4)
	FREQ	F5.0		Frequency for volume attenuation calculation
	KIND	I3	0 1 2	Printout units - KYDS - KM - NM
	IPRL		1	Line printer plot of PL vs Range. Max number of range pts is 111.
*	VBPR (I)	F4.2		Source beam pattern (dB units) at 1° intervals. If INVBP = 1 max number of values is 180 (symmetric). If INVBP = 2 max number of values is 360. Negative value terminates input.
*	BTL (I)	F4.2		Required if IABOT = 1. Bottom loss (dB) at 1° intervals. Terminate input list with negative value (I must be greater than 9).
*	FMT (I)	12A4		Required if IXPR = -2. Convolved output heading.
*	BEAM (I)	F4.2		Required if IXPR = -2. Values specified at angle increments of DELT (Sub-routine TRACE). Terminate input list with negative value.
*	SD RCD (I)	F8.1 F8.1		Source depth Receiver depths (number of range pts x number of receivers less than 4000). Terminate list with null value.
*	MORE	I5	Non Zero 0	Read new input from *A (i.e. use existing profile). Terminate run

NOTES: (1) *represents new card(s)

APPENDIX II

RAY THEORY RELATIONSHIPS

Some of the ray theory relationships used to derive intensity and range values in the model are included in this section.

II.1 Sound intensity

The intensity (I_r) at range r relative to the intensity at unit range (I_o) is given by (e.g. ref.12, page 57)

$$(I_r/I_o) = \frac{\cos\alpha_s}{r \sin\alpha_r} \cdot \left(\frac{\Delta r}{\Delta\alpha_s}\right)^{-1} \quad \text{(II.1)}$$

where α_s and $\alpha_s + \Delta\alpha_s$ are the grazing angles at the source of the pair of rays, α_r is the mean angle at the receiver and Δr is the horizontal separation of the rays at the receiver.

If the receiver is bracketted by two rays which are almost horizontal (i.e. near a ray vertex where sin α_r approaches zero) the following expression is used:

$$I_r/I_o = \frac{\cos\alpha_s}{r\Delta z} \cdot \Delta\alpha_s \quad \text{(II.2)}$$

where Δz is the vertical ray separation.

In the vicinity of a caustic the energy at the caustic is smoothed between the two rays, which bound both the receiver and the caustic.

II.2 Range calculations

Expressions will be given for calculating the horizontal range (r) travelled by a ray, which passes downwards through a layer of the ocean with values of depth and sound speed given by (z_1, c_1) and (z_2, c_2), where z_2 is greater than z_1, and c_2 is greater then c_1. Using appropriate changes of variable the case where c_1 greater than c_2 and the case where the ray turns over within a layer (c_2 then becomes equal to c_m) can be handled by the expressions given below.

The range is given by (e.g. ref.12 page 54):

$$r = \int_{z_1}^{z_2} \frac{c(z)/c_m}{(1 - c^2(z)/c^2_m)^{\frac{1}{2}}}\, dz \quad \text{(II.3)}$$

where c_m is the sound speed at the ray vertex.

Replacing the variable z, r can be rewritten:

$$r = +\Delta z \int_0^1 \frac{1}{\left[\left(\frac{c_m}{c_x}\right)^2 - 1\right]^{\frac{1}{2}}} dx \qquad \text{(II.4)}$$

where $\Delta z = z_2 - z_1$ and $x = \frac{z - z_1}{\Delta z}$

II.2.1 Linear segments

For the case of linear segments in the profile, the profile in the layer becomes:

$$C_m^2/C_x^2 - 1 = A - Bx \qquad \text{(II.5)}$$

Substituting II.5 in II.4 the range in the layer becomes:

$$r = \frac{2\Delta z}{B} \left[A^{\frac{1}{2}} - (A-B)^{\frac{1}{2}}\right] \qquad \text{(II.6)}$$

where $A = \frac{C_m^2}{C_1^2} - 1$

and $B = \frac{C_m^2}{C_1^2} - \frac{C_m^2}{C_2^2}$

II.2.2 Spline fit

$$\text{Set } y = \left[\left(\frac{C_m}{Cx}\right)^2 - 1\right]$$

Equation II.4 becomes: $r = \Delta z \int_0^1 \frac{dx}{(y(x))^{\frac{1}{2}}}$

Using Gaussian quadrature (or Gaussian integration) the intregral can be evaluated as:

$$r = \frac{\Delta z}{2} \sum_{i=1}^{n} W_i \, (y(x_i))^{-\frac{1}{2}}$$

The coefficients W_i are given for a particular value of n (e.g. ref.13) and the (x_i) values are given by: $x_i = \frac{1}{2}k_i + \frac{1}{2}$ where k_i is the i th zero of the Legendre polynomial $P_n(k)$.

The programme has included as data, values of W_i and x_i for n equal to 3. An early investigation carried out with the programme showed that this order gave adequate accuracy for the range calculation.

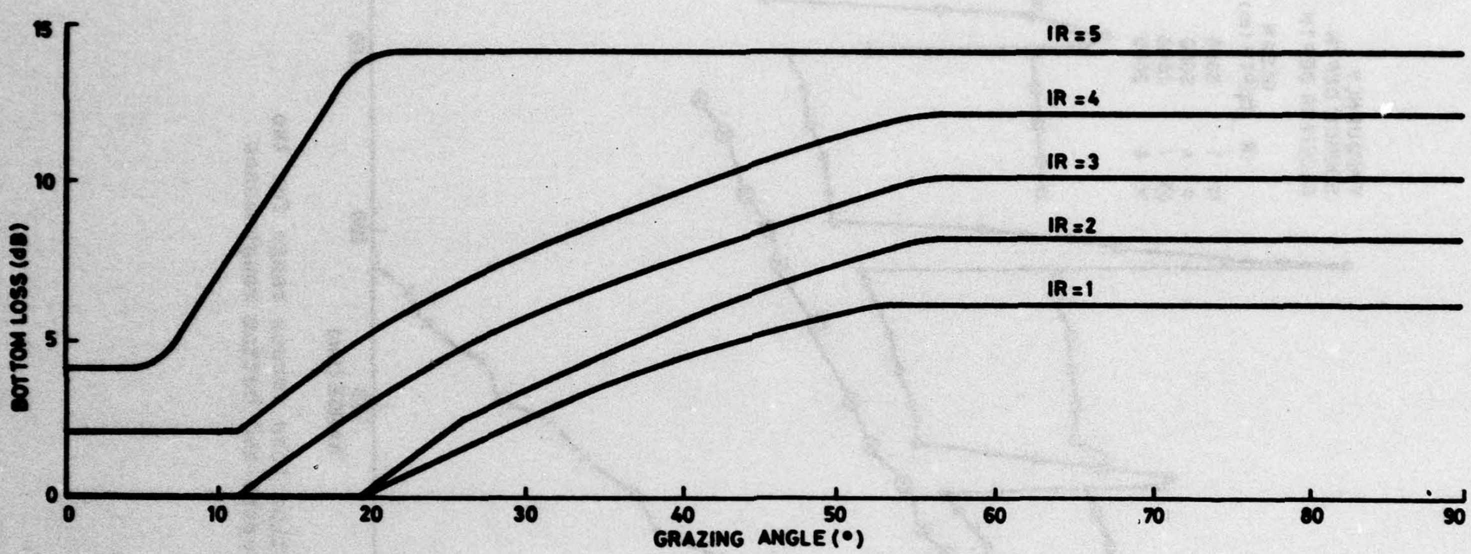


Figure 1. Bottom loss versus grazing angle for various bottom roughnesses (IR = 1 to 5) and for frequencies less than 150 Hz(ref.7)

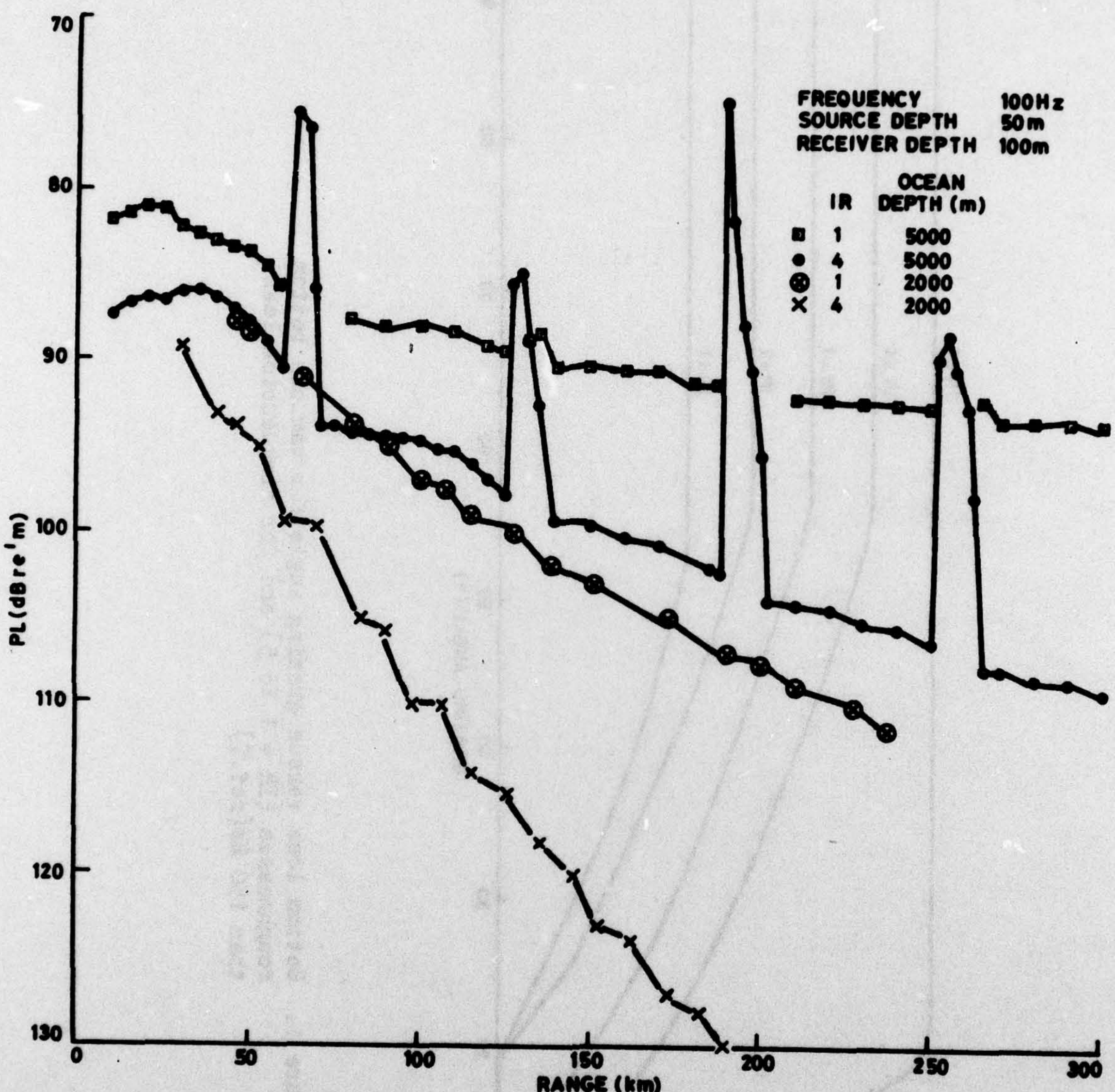


Figure 2. Propagation loss versus range for two ocean depths and bottom roughnesses

DISTRIBUTION

EXTERNAL

	Copy No.
In United Kingdom	
Defence Scientific and Technical Representative, London	1
In United States	
Counsellor Defence Science, Washington	2
In Australia	
Chief Defence Scientist	3
Executive Controller, Australian Defence Scientific Service	4
Controller, Military Studies and Operational Analysis Division	5
Navy Scientific Adviser	6
Superintendent, Central Studies Establishment	7
Director Joint Intellligence Organisation (DDSTI)	8
Superintendent, Defence Science Administration Division	9
Superintendent, R.A.N. Research Laboratory	10
Dr. J. Ranicar, Sonar Group	11
Mr. P. Nysen, Sonar Group	12
Defence Information Services Branch (for microfilming)	13 - 14
Defence Information Services Branch for:	
United States Department of Defense, Defense Documentation Center	15 - 26
United Kingdom, Ministry of Defence, Defence Research Information Centre (DRIC)	27
Canada, Department of National Defence, Defence Science Information Service	28
New Zealand, Department of Defence	29
Australian National Library	30
Defence Library, Campbell Park	31
Library, Aeronautical Research Laboratories	32
Library, Materials Research Laboratories	33

INTERNAL TO DRCS

Chief Superintendent, Weapons Systems Research Laboratory	34
Superintendent, Propulsion and Marine Physics Division	35
Senior Principal Research Scientist, Marine Studies Composite	36
Principal Officer, Marine Physics Group	37
Principal Officer, Underwater Detection Group	38
Dr. G. Gartrell, Marine Physics Group	39
Dr. A. Burgess, Underwater Detection Group	40

	Copy No.
Author	41
DRCS Library	42 - 43
PMD Library	44 - 45
Spares	46 - 50